Earth-Venus-Mars 1968-9

The advanced earth-venus-mars free-fall trajectories of this period require relatively close venus passing distances. Thus the calculations were aimed at determining the maximum distances of closest approach. It was found that these trajectories could be determined reasonably well from the fine net calculation. The results appear in Table 11. These maximum venus passing distance trajectories require approximately 6% to 12% more launch energy than the minimum (which occures when the venus closest approach distances are all zero). It is interesting to note that the distances of closest approach together with the required launch energies decrease monotonically as one proceeds through the launch period. We also notice that the earth-venus transfers are Type I requiring only about 100 days but the venus-mars transfers are Type II and require about 400 days. Thus the total mission flight times are approximately 500 days.

Let us now refer back to Table 3. We find that the launch dates for our advanced trajectories occure approximately 2 months before the launch dates for direct earth-mars 1969 Type I trajectories and occure about $2\frac{1}{2}$ months before the Type II trajectories. The mars intercept dates for the advanced missions occure 8 to 9 months after the Type I direct flight intercept dates and about $3\frac{1}{4}$ to 5 months after the Type II trajectories. Consequently we are forced to conclude that if the primary objective is to reach mars during the time interval under consideration, the direct flight trajectories of 1969 should be employed. This is also evident by the fact that a vehicle approaching mars on an advanced trajectory will have approximately 9 times as much energy relative to mars than the minimum approach energy of the direct flight trajectories. Thus we present these trajectories with the opinion that their use will probably be more valuable in the perfection of the extremely accurate planetary approach guidance system necessary to perform these advanced missions.

Table 11

EARTH-VENUS- MARS

1968-9

LAUNCH DATE	HEV_1	T_{12}	θ_{12}	$\vec{B}\cdot\vec{T}$	$\vec{B}\cdot\vec{R}$	HEV_2	TISI	DOCA	VACA	DA	T_{23}	θ_{23}	HEV_3	TFT
Dec. 22	4.98	114.55	119.78	-10580.	-1685.	8.45	1.64	989.	12.77	46.03	389.98	240.56	10.97	504.53
24	4.94	112.75	118.08	-10435.	-1707.	8.52	1.63	908.	12.86	45.86	391.52	241.08	10.99	504.28
26	4.86	111.35	117.01	-10356.	-1743.	8.50	1.63	830.	12.89	46.31	392.51	240.89	10.98	503.87
28	4.77	109.96	115.94	-10280.	-1775.	8.49	1.64	756.	12.91	46.75	393.48	240.69	10.98	503.43
30	4.71	108.36	114.55	-10181.	-1797.	8.51	1.63	685.	12.97	46.90	394.69	240.83	10.98	503.05
Jan. 1	4.63	106.96	113.48	-10116.	-1824.	8.49	1.64	617.	12.99	47.35	395.60	240.59	10.98	502.56
3	4.57	105.36	112.08	-10028.	-1841.	8.51	1.63	553.	13.04	47.50	396.77	240.70	10.98	502.13
5	4.52	103.77	110.69	-9944.	-1857.	8.53	1.63	492.	13.09	47.64	397.91	240.80	10.99	501.67
7	4.44	102.37	109.62	-9896.	-1878.	8.50	1.63	435.	13.10	48.11	398.73	240.51	10.98	501.09
9	4.41	100.57	107.91	-9793.	-1886.	8.56	1.62	381.	13.17	47.94	400.11	240.95	10.99	500.68
11	4.34	99.17	106.84	-9757.	-1904.	8.53	1.63	331.	13.18	48.40	400.86	240.63	10.98	500.03
13	4.30	97.57	105.45	-9696.	-1917.	8.54	1.63	285.	13.21	48.55	401.89	240.67	10.98	499.46
15	4.26	95.98	104.06	-9640.	-1929.	8.54	1.63	243.	13.24	48.69	402.88	240.69	10.98	498.85
17	4.21	94.58	102.98	-9622.	-1945.	8.50	1.63	205.	13.24	49.16	403.51	240.31	10.97	498.09
19	4.19	92.98	101.59	-9579.	-1958.	8.51	1.63	171.	13.27	49.30	404.44	240.30	10.97	492.42
21	4.18	91.38	100.20	-9543.	-1971.	8.51	1.63	141.	13.28	49.44	405.33	240.27	10.97	496.71

Planetary Configuration For Earth-Venus-Mars 1968-9
(Jan 1 Trajectory)

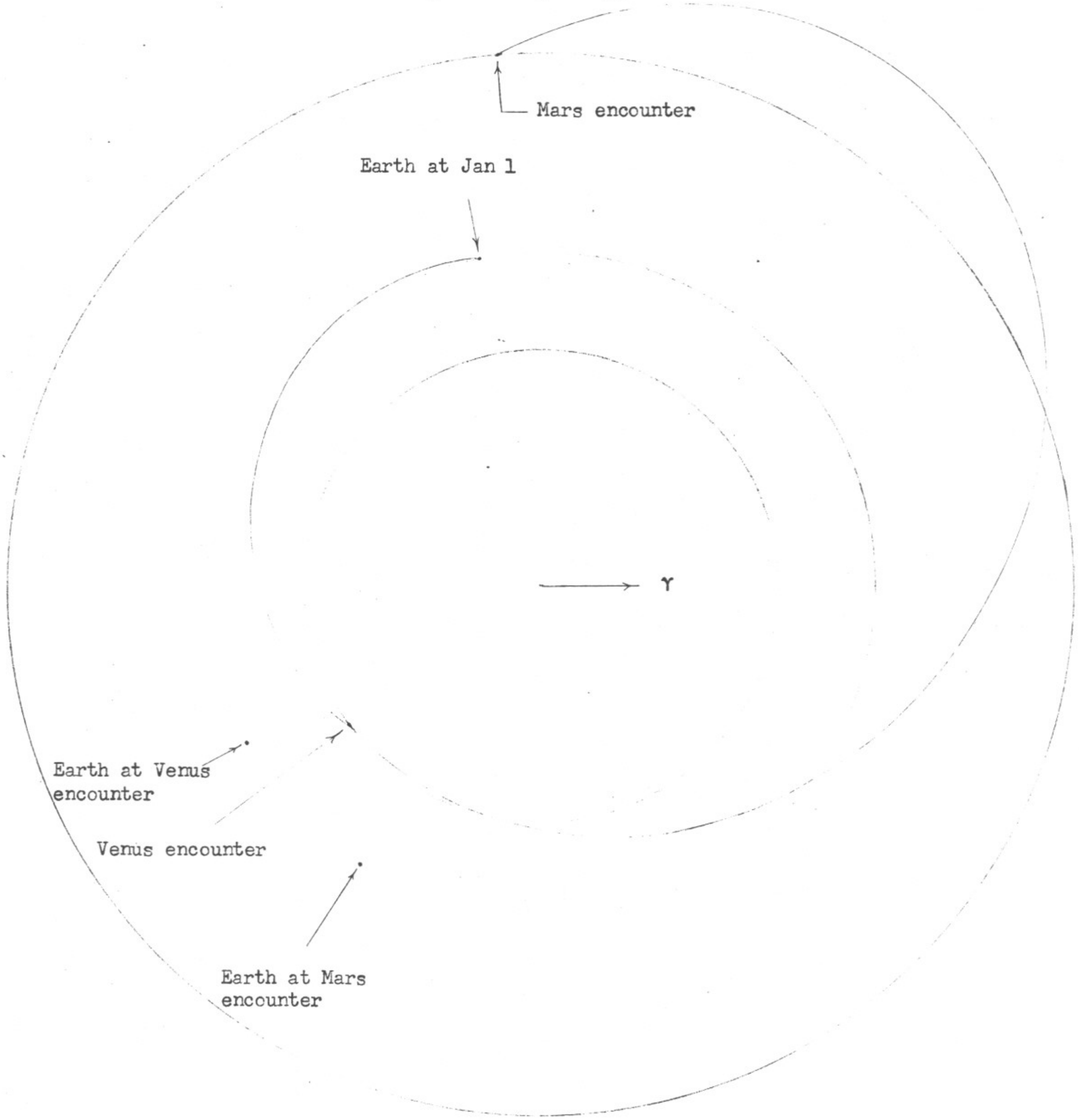


Figure 25

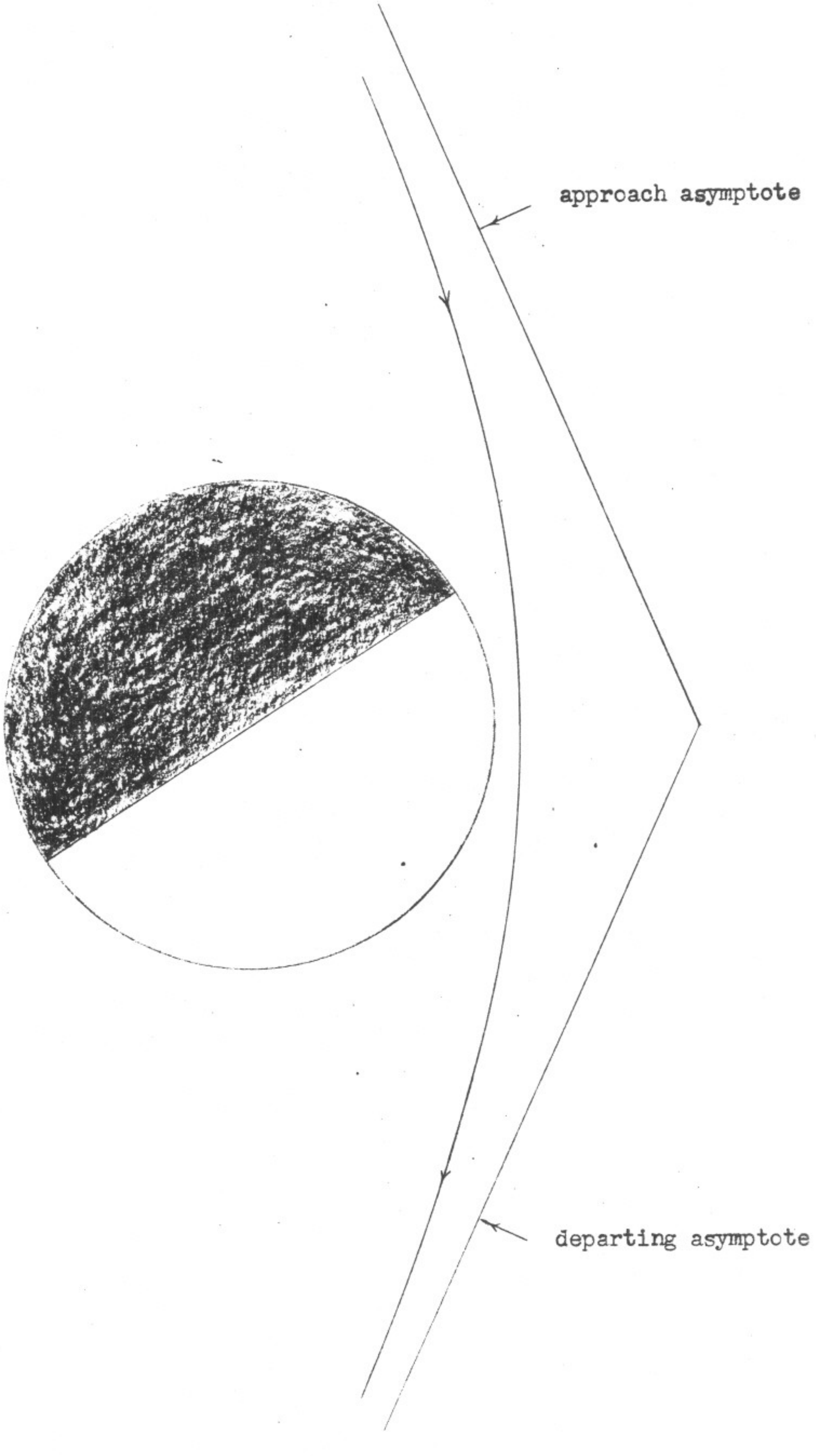


Figure 26

The planetary configuration for this period appears in figure 25. The earth's distance at the venus and mars encounters is approximately .34 A.U. and 2.5 A.U. respectively. Figure 26 describes the Jan. 1 trajectory in the immediate vicinity of venus.

Earth-Venus Mars 1970

The 1970 earth-venus-mars trajectories were found to be much more attractive than those of the previous period. An extra fine net was calculated to accurately determine the minimum launch energy trajectories. These minimum energy trajectories are given in Table 12. One immediately finds that the earth-venus transfers are almost optimum trajectories described in Table 1. The distances of closest approach are well outside venus's atmosphere. Turning to Table 3 we notice that these trajectories have flight times only about 20-40 days longer than those required for optimum Type II earth-mars trajectories for the 1971 launch period. We also notice that the launch and mars intercept dates for these advanced trajectories are almost half way between those of the optimum earth-mars 1969 and 1971 trajectories.

The planetary configuration for this launch period appears in figure 27. The earth is approximately .5 A.U. way from the vehicle at the venus encounter and approximately .62 A.U. away during the mars encounter. The trajectories with launch date August 12 in the vicinity of venus is shown in figure 28.

Earth-Venus-Mars 1972

Advanced trajectories for this period are given in Table 13. It was found that the May 11 through May 25 trajectories yield minimum launch energy when the venus closest approaches are zero. Thus these trajectories appearing in the table were chosen on the basis of low launch energy together with safe distances of closest approach. The May 27 - June 6 trajectories were found to have minimum energy when the distances of closest approach were not zero. Thus these trajectories are minimum launch energy trajectories. The trajectories appearing in the table were found from a net with grid .5 by 2. days.

TABLE 12
EARTH-VENUS-MARS
1970

LAUNCH DATE	HEV_1	T_{12}	θ_{12}	$\vec{B}\cdot\vec{T}$	$\vec{B}\cdot\vec{R}$	HEV_2	TISI	DOCA	VACA	DA	T_{23}	θ_{23}	HEV_3	TFT
July 23	3.52	142.17	159.33	-12927.	19125.	5.91	2.30	9490.	8.75	43.87	196.21	189.98	6.05	338.39
25	3.48	140.80	158.43	-14121.	19323.	5.87	2.31	10191.	8.62	43.03	196.99	189.82	5.98	337.79
27	3.44	139.45	157.57	-15407.	19537.	5.82	2.33	10986.	8.48	42.09	198.97	190.36	5.87	338.43
29	3.40	138.13	156.77	-16592.	19722.	5.78	2.34	11733.	8.36	41.28	201.95	191.47	5.74	340.09
31	3.37	136.88	156.00	-17556.	19880.	5.75	2.36	12355.	8.26	40.67	205.45	192.85	5.62	342.33
Aug. 2	3.35	135.69	155.45	-10574.	15577.	5.72	2.36	5252.	9.48	55.64	180.00	176.71	6.94	315.69
4	3.32	134.34	154.57	-10496.	15240.	5.66	2.38	4869.	9.55	57.36	180.00	176.05	6.91	314.32
6	3.29	133.00	153.75	-10496.	14956.	5.61	2.40	4571.	9.61	58.86	180.00	175.36	6.87	313.00
8	3.28	131.71	152.98	-10538.	14705.	5.56	2.42	4321.	9.66	60.22	180.00	174.63	6.84	311.71
10	3.27	130.47	152.29	-10584.	14455.	5.52	2.43	4082.	9.71	61.55	180.00	173.85	6.79	310.47
12	3.26	129.28	151.68	-10630.	14206.	5.47	2.45	3850.	9.76	62.87	180.00	173.01	6.75	309.28
14	3.28	128.15	151.18	-10651.	13939.	5.43	2.47	3603.	9.82	64.21	180.00	172.11	6.70	308.15
16	3.30	127.11	150.80	-10639.	13650.	5.40	2.48	3335.	9.90	65.61	180.00	171.13	6.65	307.11
18	3.34	126.14	150.56	-10577.	13328.	5.37	2.49	3036.	9.99	67.06	180.00	170.07	6.59	306.14
20	3.39	125.29	150.48	-10447.	12963.	5.35	2.50	2698.	10.12	68.59	180.00	168.89	6.53	305.29
22	3.47	124.55	150.61	-10226.	12540.	5.34	2.50	2305.	10.29	70.20	180.00	167.59	6.46	304.55
24	3.57	123.95	150.94	-9898.	12059.	5.36	2.49	1862.	10.50	71.85	180.00	166.16	6.39	303.95
26	3.70	123.50	151.51	-9445.	11518.	5.40	2.47	1367.	10.78	73.50	180.00	164.58	6.32	303.50
28	3.86	123.18	152.29	-8864.	10931.	5.48	2.44	835.	11.12	75.06	180.00	162.86	6.24	303.18

Planetary Configuration For Earth-Venus-Mars 1970 (August 12 Trajectory)

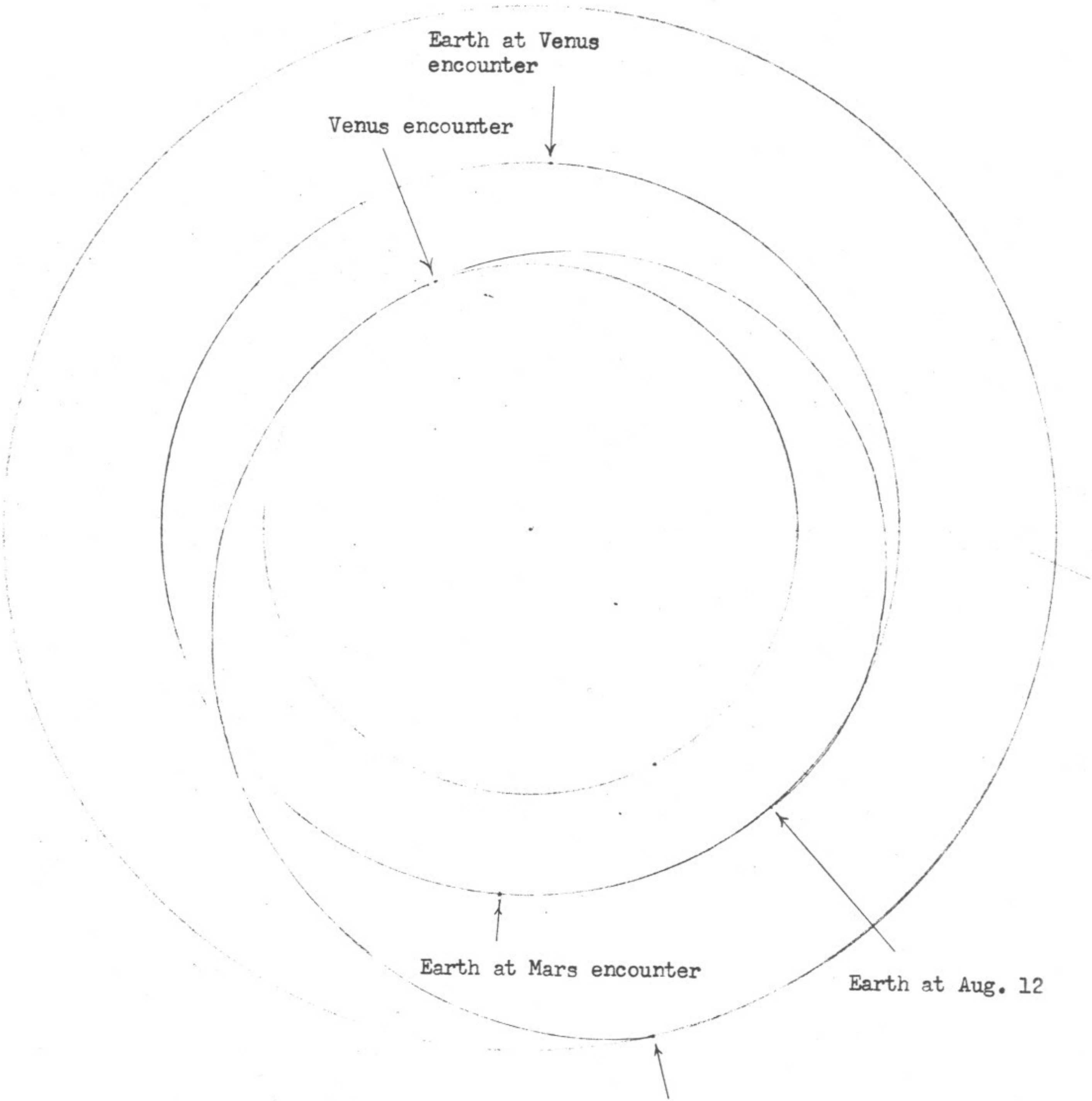


Figure 27

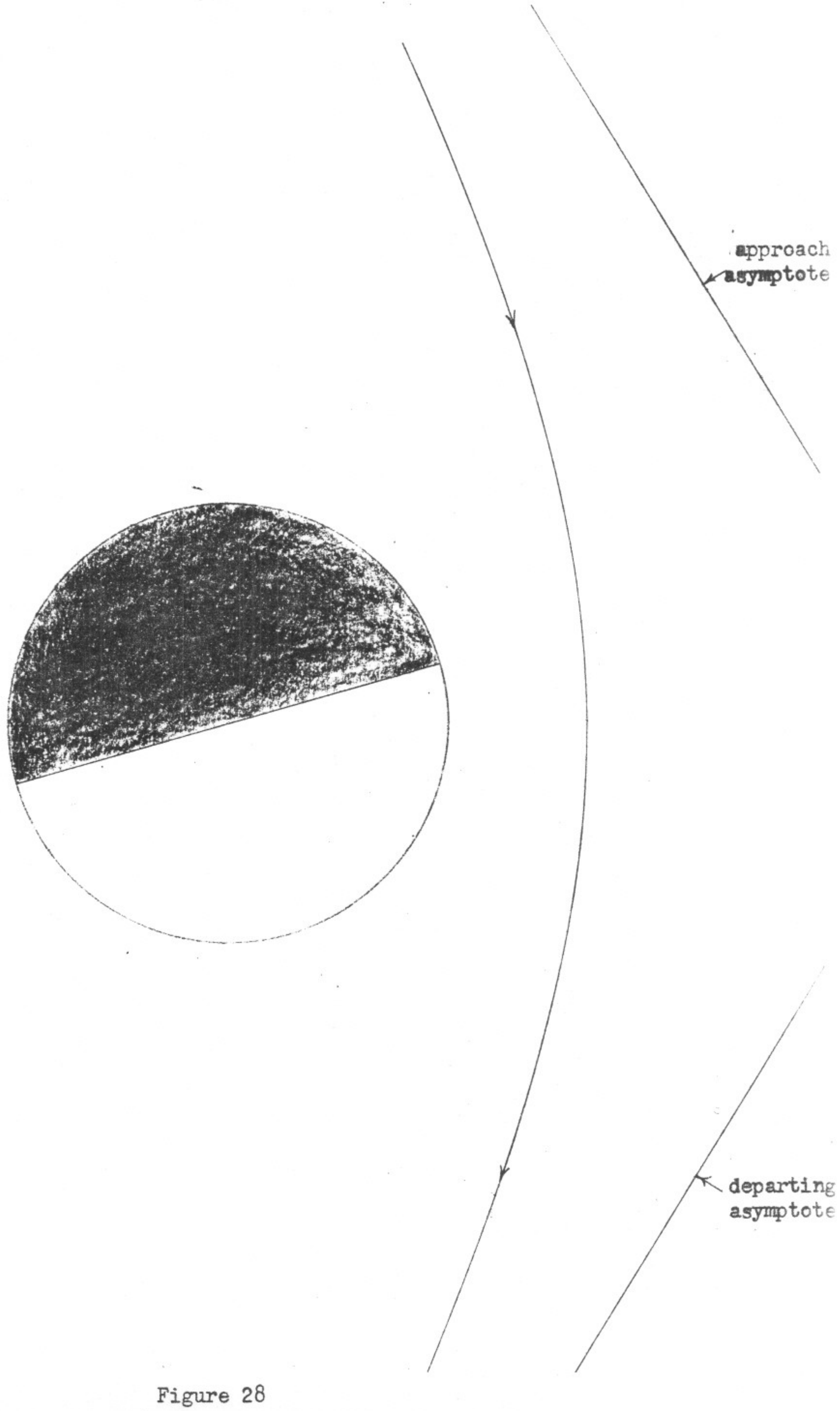


Figure 28

Table 13

EARTH-VENUS-MARS

1972

LAUNCH DATE	HEV_1	T_{12}	θ_{12}	$\vec{B}\cdot\vec{T}$	$\vec{B}\cdot\vec{R}$	TISI	DOCA	VACA	DA	T_{23}	θ_{23}	HEV_3	TFT
May 11	4.33	183.50	269.68	9991.	-75.	1.59	490.	13.21	46.36	113.31	108.78	13.24	296.81
13	4.22	181.00	266.94	10176.	-150.	1.61	581.	13.07	46.86	114.45	109.94	13.09	295.45
15	4.10	178.00	263.39	10041.	-261.	1.64	354.	13.09	49.10	114.87	111.25	13.18	292.87
17	4.04	175.50	260.65	10076.	-340.	1.66	325.	13.05	49.96	155.50	112.13	13.14	291.00
19	4.02	173.50	258.73	10302.	-402.	1.67	507.	12.92	49.39	116.34	112.58	12.97	289.84
21	4.03	172.00	257.62	10794.	-454.	1.66	966.	12.67	47.26	117.70	112.77	12.61	289.70
23	4.04	170.00	255.70	10958.	-533.	1.67	1103.	12.59	46.86	118.31	113.10	12.49	288.31
25	4.0	16.0	25	1	-6	1.6		12		11	113	1	28
27	4.11	165.50	251.05	10757.	-710.	1.68	887.	12.66	48.21	117.98	113.47	12.64	283.48
29	4.16	165.50	252.38	12590.	-817.	1.65	2691.	11.98	40.52	123.47	114.29	11.29	288.97
31	4.22	165.00	252.91	14802.	-875.	1.64	4869.	11.44	34.04	132.56	117.70	9.66	297.56
June 2	4.27	165.00	254.24	18083.	-220.	1.61	8123.	10.94	27.27	152.76	127.05	7.25	317.76
4	4.32	165.00	255.57	19229.	1138.	1.59	9383.	10.89	24.83	170.13	135.17	6.19	335.13
6	4.37	165.00	256.91	19022.	2470.	1.56	9413.	11.00	24.14	182.72	140.69	5.96	347.72

Planetary Configuration For Earth-Venus-Mars 1972
(May 21 Trajectory)

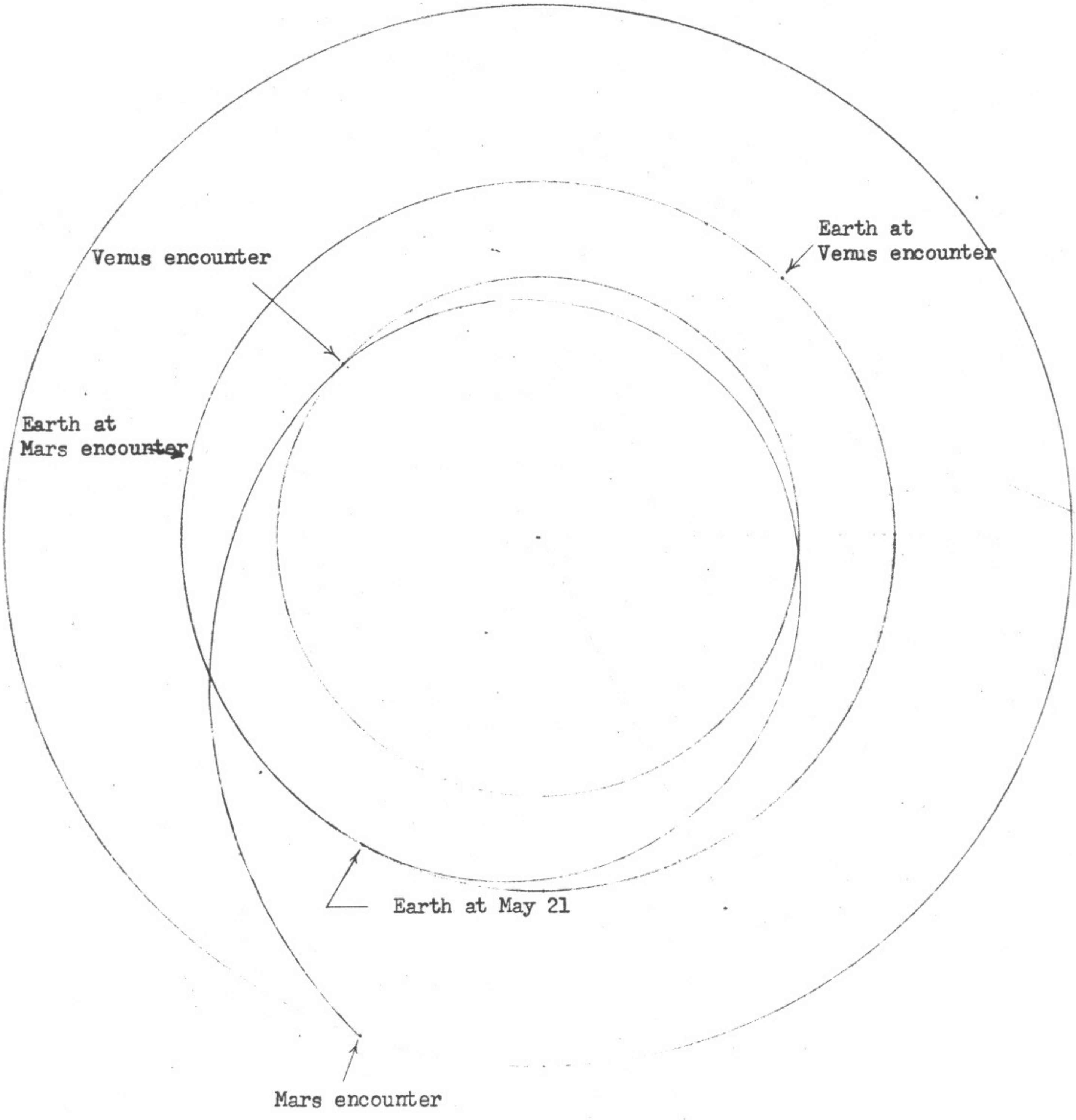


Figure 29

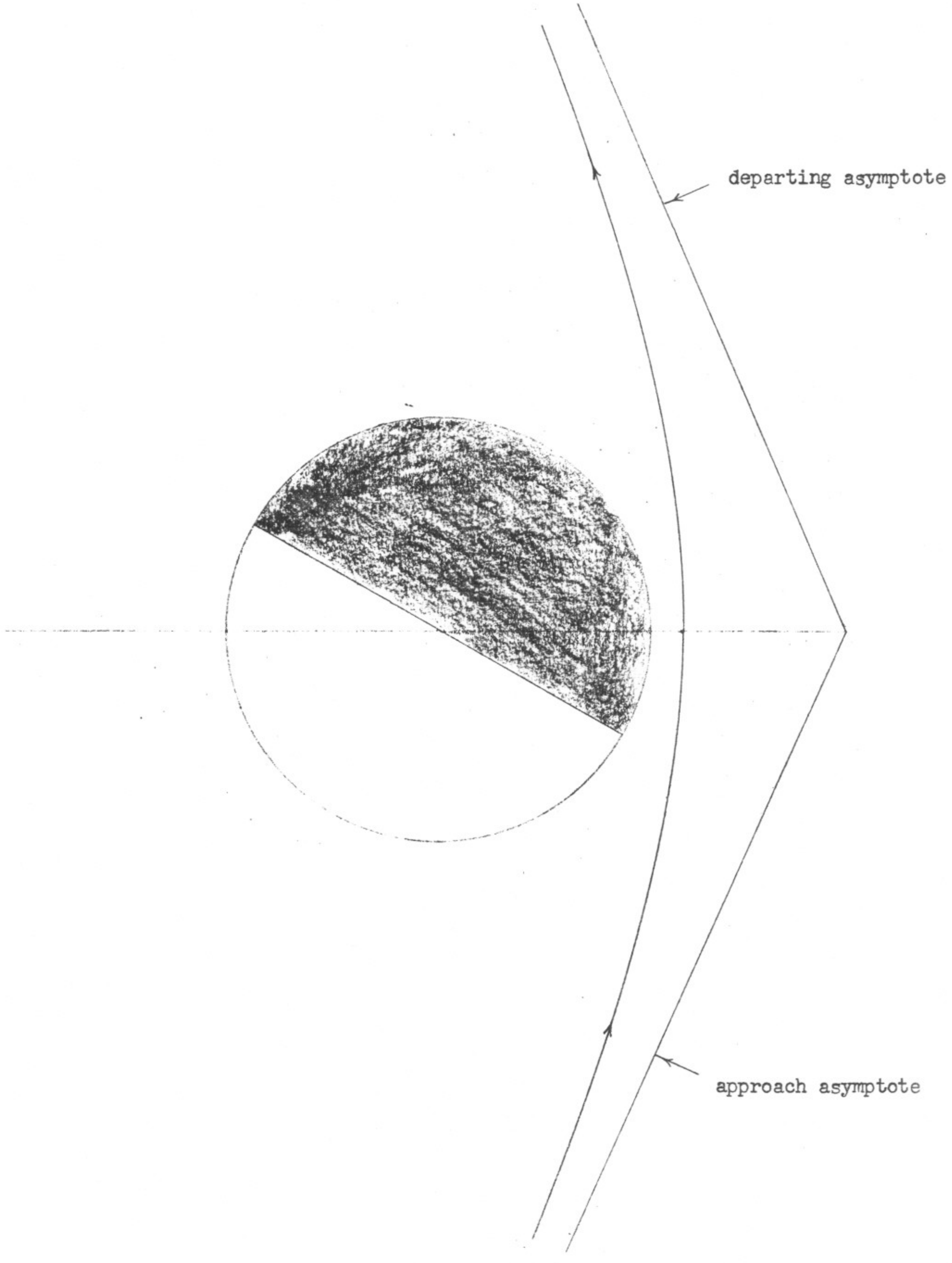


Figure 30

Refering to Table 3 one finds that this launch opportunity falls very conveniently right between the 1971 and 1973 earth-mars trajectories. Indeed we notice that they are even better than the Type II trajectories for 1973. Figure 29 shows the over-all geometric aspect of these trajectories while figure 30 describes the May 21 trajectory in the immediate vicinity of venus.

The numerical results given in tables 4 through 13 definitely indicate that space vehicles equipted with a highly accurate planetary approach guidance system can play an important roll in the unmanned exploration of mercury and mars.

B. Initial Interplanetary Missions by Manned Vehicles

We now consider possible applications of advanced free-fall trajectories to manned interplanetary space flights. Perhaps the first manned interplanetary voyages will be simple reconnaissance missions of venus and mars without actually landing or going into orbit about these planets. Since our study is confined to the decade beginning in 1965 we shall confine ourselves to the period 1970 to 1975. Let us now turn to the first of these two types of reconnaissance missions.

a. Earth-Venus-Earth 1970-1974

The earth-Venus-earth missions were found to be much more reasonable than the earth-mars-earth missions. It was discovered that the former have launch periods (for the favorable trajectories) which almost spands the entire launch periods for the one way earth-venus trajectories given in Table 1. Thus relatively coarse nets were found sufficient to obtain a general idea of the characteristics of the most favorable trajectories for each period. Table 13 is a selection of 3 trajectories from the 3 launch periods in 1970-1975. They were chosen on the basis of short flight times, low launch energy and of course safe distances of closest approach. The first was obtained from a 2 by 2 day net, while the second and third came from 6 by 6 day nets.

Now the Nova launch vehicle which may be ready by 1970 is expected to be able to place 248k or 228k on escape trajectories having hyperbolic excess velocitites of approximately 3 and 4 km/sec respectively. The appollow space vehicle is expected to weigh about 25k. Thus the trajectories given in Table 13 show that it may be possible to carry out these manned venus reconnaissance missions using the Nova launch vehicle. This possibility will be taken up later.

We turn now to the manned earth-mars-earth reconnaissance missions.

b. Earth-Mars-Earth 1970-1974

There are only 2 earth-mars launch periods in 1970-1974. These occure in 1971 and 1973 (see Table 3). Both were analysed for possible earth-mars-earth trajectories by calculating coarse nets with grids 4 by 4 and 6 by 8 days respectively. The results did not indicate that a more detailed study was necessary. All of the trajectories were found to require high launch energies and all had flight times in excess of 1000 days. The best trajectories found from each net appear in Table 14. The long flight times result from Type III mars-earth transfers. It is interesting to notice that the 1973 trajectory permits the vehicle to remain under the influence of mars for almost $5\frac{1}{2}$ days. This is due to the fact that the vehicle's hyperbolic excess velocity as it approaches mars is only 2.53 km/sec. The long flight times required for these trajectories make these missions impractical.

Thus with respect to the manned planetary reconnaissance missions we find ourselves faced with another very sad situation. The Nova could probably be used as the launch vehicle for manned venus reconnaissance missions but not for manned mars reconnaissance missions during the time period 1970-1974.

During the latter part of June 1962 while the author was checking some newely calculated advanced trajectories of the form earth-venus-mars-earth, a very remarkable fact was discovered. Now it was already known at that time that the earth-mars-earth trajectories had very long flight times. Thus it was believed

TABLE 13

Earth - Venus - Earth

Launch Date	HEV_1	T_{12}	θ_{12}	$\vec{B}\cdot\vec{T}$	$\vec{B}\cdot\vec{R}$	TISI	DOCA	VACA	DA	T_{23}	θ_{23}	HEV_3	TFT
8/20/70	2.92	114.00	132.23	-13653.	-3187.	2.46	725.5	11.29	76.16	250.96	227.48	7.13	364.96
4/3/72	3.69	114.00	134.07	-18362.	4964.	2.69	3983.	9.47	68.29	260.95	235.43	8.18	374.95
11/4/73	3.76	110.00	139.29	-22410.	219.	2.94	5343.5	8.76	71.80	269.46	241.00	7.90	385.46

TABLE 14

Earth - Mars - Earth

Launch Date	HEV_1	T_{12}	θ_{12}	$\vec{B}\cdot\vec{T}$	$\vec{B}\cdot\vec{R}$	TISI	DOCA	VACA	DA	T_{23}	θ_{22}	HEV_2	TFT
6/8/71	3.97	316.00	204.84	1017.	7652.	3.23	2251.1	5.74	34.88	795.83	530.43	5.85	1111.83
8/20/73	4.60	236.00	151.68	2222.	15623.	5.46	7024.	3.83	46.06	790.72	502.25	6.56	1027.7

that the most favorable earth-venus-mars-earth trajectories would have flight times much longer than 1000 days. The fact is, however that this is not always true. It was discovered that in some cases this assumption was false by a very wide margin. These cases very conveniently turned out to be the 1970 and the 1972 earth-venus launch periods.

c. Earth-Venus-Mars-Earth 1970

These manned venus-mars reconnaissance trajectories were studied very carefully. Three nets were calculated. The first net had a grid of 3 by 3 days while the second and third were fine and extra fine nets with grids .2 by 2 and .01 by 2 days respectively. It was found that the minimum launch energy trajectories had ideal distances of closest approach and almost minimum flight times. These trajectories are given in Table 15. Recalling the near optimum earth-mars-earth trajectories in Table 14 we notice the remarkable fact that these more complicated trajectories require only about one half of the launch energies and flight times required by the best earth-mars-earth reconnaissance trajectories of the 1971 and 1973 periods. It should be pointed out however that these trajectories will bring a space vehicle back to earth with a great deal of energy. If this high return energy problem can be solved without employing massive retro rockets these missions could probably be accomplished with a Nova type launch vehicle. These venus-mars reconnaissance missions offer a very attractive first step in manned interplanetary space flights. They could be extremely useful in paving the way for actual manned landings on venus and mars.

The planetary configuration for these manned reconnaissance missions appears in figure 31. We notice that the vehicle is never very far from the earth throughout the entire flight. Thus continuous radio communication should be readily available. It is also important to notice that the entire trajectory is confined to the region between the orbits of venus and mars. The venus and mars encounters takes place

TABLE 15

Earth-Venus-Mars-Earth 1970

Launch Date	HEV_1	T_{12}	θ_{12}	$(B\cdot T)_2$	$(B\cdot R)_2$	HEV_2	$TISI_2$	$DCCA_2$	$VACA_2$	DA_2	T_{23}	θ_{23}
July 15	3.74	147.86	163.23	-9238.	18335.	6.11	2.23	7495.	9.22	45.95	196.99	192.81
17	3.68	146.42	162.22	-10626.	19898.	6.06	2.25	9278.	8.88	42.87	210.00	200.13
19	3.63	144.99	161.24	-11499.	20089.	6.01	2.26	9732.	8.78	42.52	210.00	199.55
21	3.57	143.58	160.28	-11806.	18872.	5.96	2.28	8819.	8.89	44.68	195.68	190.20
23	3.52	142.18	159.34	-12883.	19081.	5.91	2.30	9435.	8.76	43.96	195.81	189.74
25	3.48	140.80	158.44	-14114.	19312.	5.87	2.31	10179.	8.62	43.05	196.88	189.75
27	3.44	139.45	157.57	-15406.	19531.	5.82	2.33	10981.	8.48	42.10	198.92	190.32
29	3.40	138.14	156.77	-16592.	19718.	5.78	2.34	11731.	8.36	41.28	210.85	191.40
31	3.37	136.88	156.06	-17570.	19867.	5.75	2.36	12355.	8.26	40.67	205.30	192.75
Aug. 2	3.35	135.69	155.46	-10571.	15575.	5.72	2.26	5250.	9.48	55.65	180.00	176.71
4	3.32	134.33	154.58	-10493.	15237.	5.66	2.38	4867.	9.56	57.37	180.00	176.05
6	3.29	133.00	153.75	-10494.	14955.	5.61	2.40	4569.	9.61	58.86	180.00	175.36
8	3.28	131.71	152.98	-10536.	14703.	5.56	2.42	4320.	9.66	60.22	180.00	174.63
10	3.27	130.47	152.29	-10583.	14453.	5.52	2.43	4080.	9.71	61.56	180.00	173.84
12	3.26	129.28	151.68	-10628.	14205.	5.47	2.45	3848.	9.76	62.87	180.00	173.01
14	3.28	128.16	151.18	-10649.	13937.	5.43	2.47	3601.	9.82	64.22	180.00	172.11
16	3.30	127.11	150.80	-10637.	13649.	5.40	2.48	3334.	9.90	65.61	180.00	171.13
18	3.34	126.15	150.56	-10575.	13327.	5.37	2.49	3035.	10.00	67.07	180.00	170.06
20	3.39	125.29	150.48	-10446.	12962.	5.35	2.50	2696.	10.12	68.59	180.00	168.89
22	3.47	124.56	150.61	-10224.	12539.	5.34	2.50	2304.	10.29	70.20	180.00	167.59
24	3.57	123.96	150.95	-9897.	12057.	5.36	2.49	1861.	10.50	71.85	180.00	166.16
26	3.70	123.50	151.51	-9446.	11518.	5.40	2.47	1368.	10.78	73.50	180.00	164.58
28	3.86	123.18	152.29	-8865.	10932.	5.48	2.44	836.	11.12	75.06	180.00	162.86

TABLE 15 (continued)

Launch Date	$(B\cdot T)_3$	$(\vec{B}\cdot\vec{R})_3$	HEV_3	$TISI_3$	$DOCA_3$	$VACA_3$	DA_3	T_{34}	θ_{34}	HEV_4	TFT
July 15	11551.	5981.	6.11	2.02	8494.	6.68	10.10	303.50	285.09	8.77	648.35
17	9407.	1642.	5.63	2.18	4873.	6.48	16.18	294.24	281.05	8.42	650.65
19	9474.	1468.	5.61	2.18	4904.	6.47	16.20	293.91	280.93	8.41	648.90
21	13120.	5747.	6.11	2.03	9802.	6.62	9.20	303.35	285.05	8.77	642.50
23	13724.	5037.	6.07	2.04	10083.	6.57	9.13	302.67	284.71	8.73	640.67
25	14027.	3777.	5.99	2.06	9962.	6.50	9.43	301.33	287.02	8.67	639.01
27	13659.	2225.	5.87	2.10	9235.	6.43	10.29	299.28	283.02	8.58	637.65
29	12609.	880.	5.74	2.14	7988.	6.37	11.77	296.76	281.89	8.49	636.75
31	11214.	-23.	5.62	2.18	6521.	6.35	13.82	294.21	280.93	8.42	636.38
Aug. 2	5574.	12065.	6.94	1.80	9012.	7.42	7.68	312.08	292.15	9.51	630.77
4	5103.	11317.	6.91	1.81	8132.	7.43	8.30	314.63	291.95	9.48	628.96
6	4844.	10806.	6.87	1.82	7552.	7.42	8.79	314.13	291.72	9.45	627.13
8	4704.	10430.	6.84	1.82	7143.	7.41	9.19	313.58	291.46	9.42	625.29
10	4649.	10121.	6.79	1.83	6830.	7.39	9.56	313.00	291.18	9.38	623.47
12	4644.	9870.	6.75	1.85	6590.	7.36	9.89	312.36	290.86	9.34	621.63
14	4684.	9646.	6.70	1.86	6393.	7.32	10.20	311.66	290.51	9.30	619.81
16	4762.	9437.	6.65	1.87	6227.	7.29	10.52	310.89	290.13	9.26	618.00
18	4877.	9227.	6.59	1.89	6079.	7.24	10.83	310.06	289.71	9.21	616.21
20	5026.	9006.	6.53	1.90	5939.	7.20	11.17	309.15	289.25	9.16	614.44
22	5211.	8756.	6.46	1.92	5797.	5.15	11.53	308.15	288.75	9.10	612.70
24	5427.	8465.	6.39	1.94	5643.	7.09	11.95	307.05	288.21	9.04	611.01
26	5670.	8115.	6.32	1.96	5466.	7.04	12.42	305.87	287.63	8.98	609.37
28	5926.	7682.	6.24	1.98	5246.	6.99	12.98	304.65	287.06	8.92	607.82

Planetary Configuration For Earth-Venus-Mars-Earth 1970; Aug. 12 trajectory

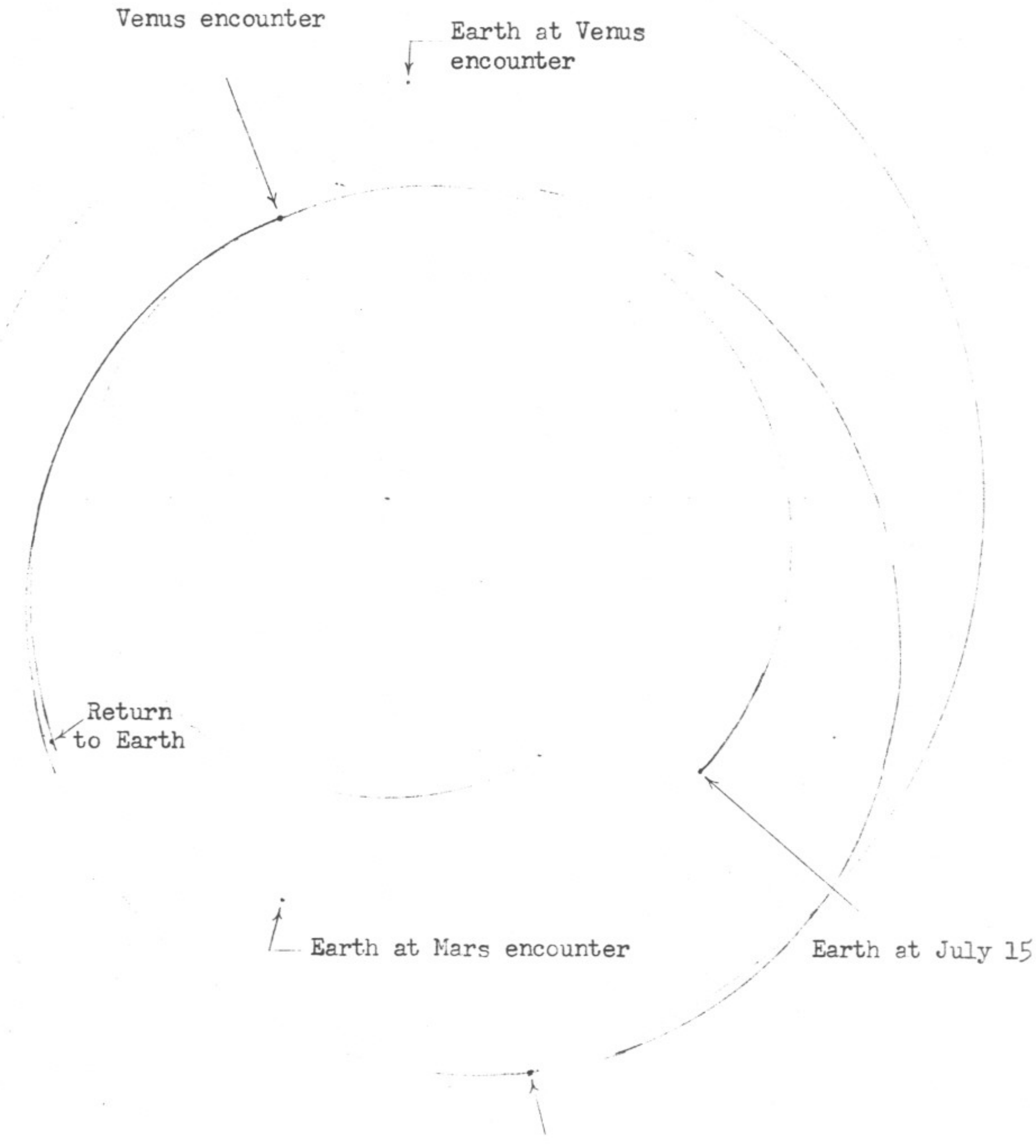


Figure 31

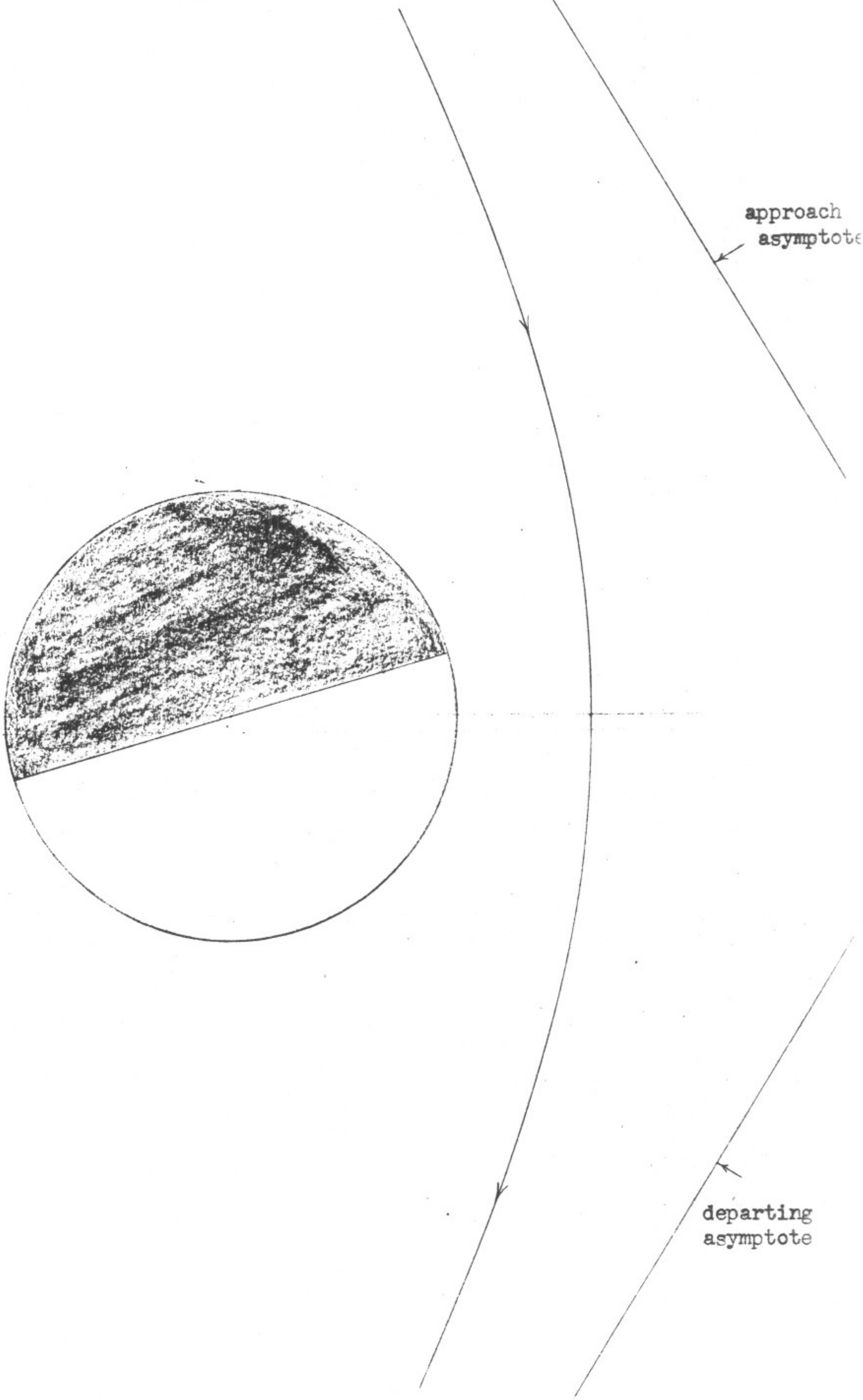


Figure 32

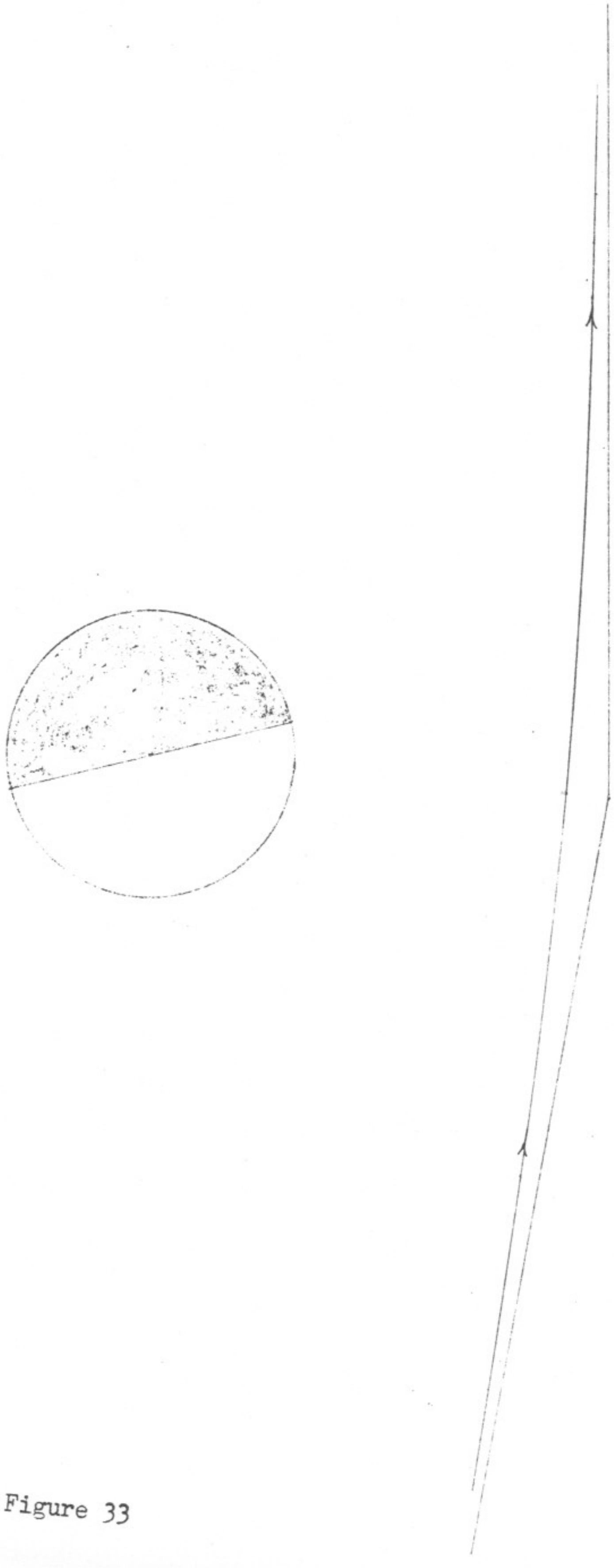


Figure 33

when the vehicle is approximately .35 and .6 A.U.'s from the earth. Figures 32 and 33 show the August 12 trajectory in the vicinity of venus and mars respectively. These figures are drawn to the same scale. According to this scale the radius of the gravitational sphere of influence of venus is 12.8 ft. while the radius of mars' sphere of influence is 11.2 ft.

Earth-Venus-Mars-Earth 1972

The advanced venus-mars reconnaissance trajectories for this period turned out to be even more surprising than those of the previous period. It was observed that trajectories existed which had much shorter flight times than those of the 1970 period. These short flight time trajectories also occured when the launch energies were nearly minimum. It was found that the flight times were minimum when the mars distances of closest approach were zero. Thus our search was directed toward finding trajectories which had near minimum flight times but not passing too close to the surface of mars. The fine net calculation proved to be sufficient for this prupose. The trajectories of this net which were selected are given in Table 16. Comparing these trajectories with those of the 1970 period we find that the flight times are about 150 days shorter. The launch energies however are about 60% greater while the return energies are about 200% greater. By increasing the launch energies and flight times by a small amount it is possible to reduce these high return energies. The reason for these short flight times is due to the fact that the venus-mars and mars-earth transfers are both Type I while the earth-venus transfers are Type II. For the 1970 trajectories this situation is reversed.

Although the energy requirements for these trajectories are somewhat higher than those of the previous period these missions are probably still within the capabilities of a Nova. The planetary configuration for these manned reconnaissance missions appears in figure 34. Figures 35 and 36 describes the May 27 trajectory as it passes venus and mars.

TABLE 16

EARTH - VENUS - MARS - EARTH 1972

Launch Date	HEV_1	T_{12}	θ_{12}	$(\bar{B}\cdot\bar{T})_2$	$(\bar{B}\cdot\bar{R})_2$	HEV_2	$TISI_2$	$DOCA_2$	$VACA_2$	DA_2	T_{23}	θ_{23}
May 13	4.73	186.93	276.57	15319.	1435.	9.31	1.49	5988.	11.85	27.38	137.98	116.57
15	4.56	184.34	273.67	15674.	1278.	9.13	1.52	6207.	11.67	27.82	139.62	118.12
17	4.42	181.74	270.78	15876.	1046.	8.97	1.55	6282.	11.53	28.44	140.22	119.09
19	4.32	179.35	268.22	16156.	876.	8.86	1.57	6462.	11.41	28.70	141.50	120.24
21	4.24	176.95	265.65	16303.	646.	8.76	1.58	6521.	11.32	29.10	141.91	120.89
23	4.19	174.55	263.09	16308.	364.	8.67	1.60	6455.	11.27	29.67	141.45	121.05
25	4.17	172.35	260.85	16457.	156.	8.62	1.61	6553.	11.21	29.78	142.04	121.59
27	4.16	170.16	258.61	16491.	-93.	8.57	1.62	6552.	11.17	30.01	141.94	121.74
29	4.18	168.16	256.70	16723.	-272.	8.55	1.62	6766.	11.12	29.73	143.16	122.43
31	4.22	165.96	254.47	16523.	-588.	8.53	1.62	6561.	11.14	30.22	141.68	121.80
June 2	4.27	163.97	252.56	16543.	-834.	8.53	1.62	6592.	11.13	30.15	141.65	121.78
4	4.34	161.97	250.65	16457.	-1103.	8.54	1.62	6532.	11.15	30.21	141.04	121.43
6	4.42	160.17	249.07	16648.	-1327.	8.57	1.62	6756.	11.14	29.65	142.10	121.82
8	4.51	158.37	247.49	16753.	-1578.	8.61	1.61	6907.	11.14	29.19	142.70	121.95

TABLE 16 (Continued)

Launch Date	$(\bar{B}\cdot\bar{T})_3$	$(\bar{B}\cdot\bar{R})_3$	HEV_3	$TISI_3$	$DOCA_3$	$VACA_3$	DA_3	T_{34}	θ_{34}	HEV_4	TFT
May 13	-4092.	2841.	8.53	1.50	1011.	9.60	13.52	156.56	78.25	13.36	481.4
15	-4283.	3188.	8.40	1.52	1350.	9.42	13.01	157.15	79.24	13.13	481.1
17	-4275.	3124.	8.39	1.52	1305.	9.42	13.14	157.26	79.35	13.12	479.2
19	-4437.	3459.	8.29	1.54	1620.	9.26	12.69	157.73	80.16	12.93	478.5
21	-4437.	3422.	8.28	1.54	1596.	9.26	12.76	157.80	80.24	12.91	476.6
23	-4270.	3028.	8.37	1.53	1242.	9.41	13.37	157.48	79.58	13.07	473.4
25	-4344.	3163.	8.32	1.53	1374.	9.34	13.17	157.69	79.95	12.99	472.0
27	-4283.	3026.	8.35	1.53	1249.	9.39	13.40	157.59	79.72	13.04	469.6
29	-4516.	3540.	8.22	1.55	1722.	9.18	12.65	158.14	80.75	12.80	469.4
31	-4191.	2841.	8.40	1.52	1075.	9.47	13.72	157.41	79.37	13.12	465.0
June 2	-4185.	2831.	8.40	1.52	1066.	9.48	13.74	157.40	79.35	13.13	463.0
4	-4059.	2618.	8.47	1.51	854.	9.59	14.13	157.10	78.81	13.25	460.1
6	-4317.	3095.	8.33	1.53	1314.	9.36	13.29	157.67	79.87	13.01	459.9
8	-4473.	3455.	8.25	1.55	1640.	9.22	12.76	157.99	80.51	12.86	459.0

Planetary Configuration For Earth-Venus-Mars-Earth May 27, 1972

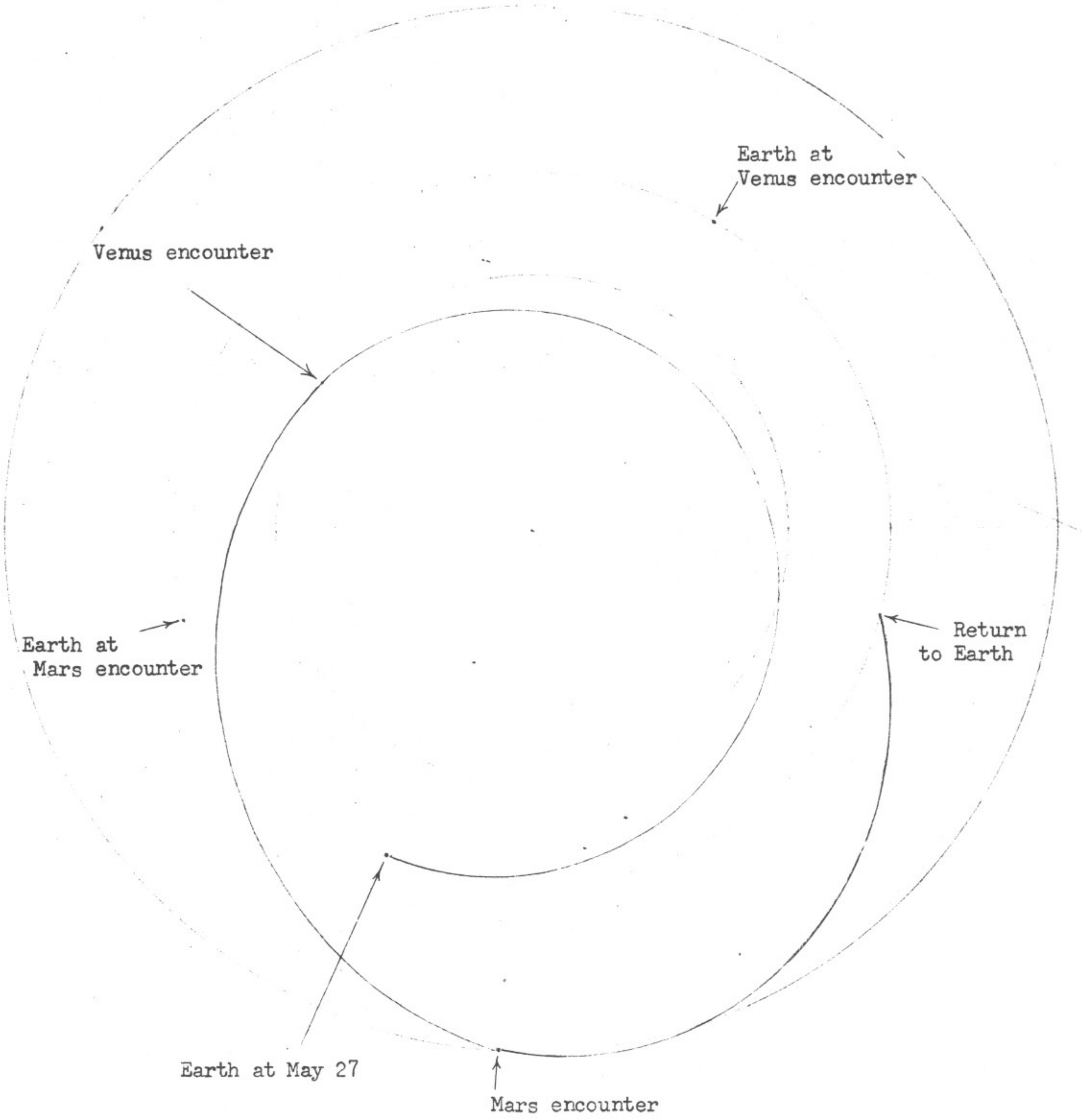


Figure 34

Figure 35

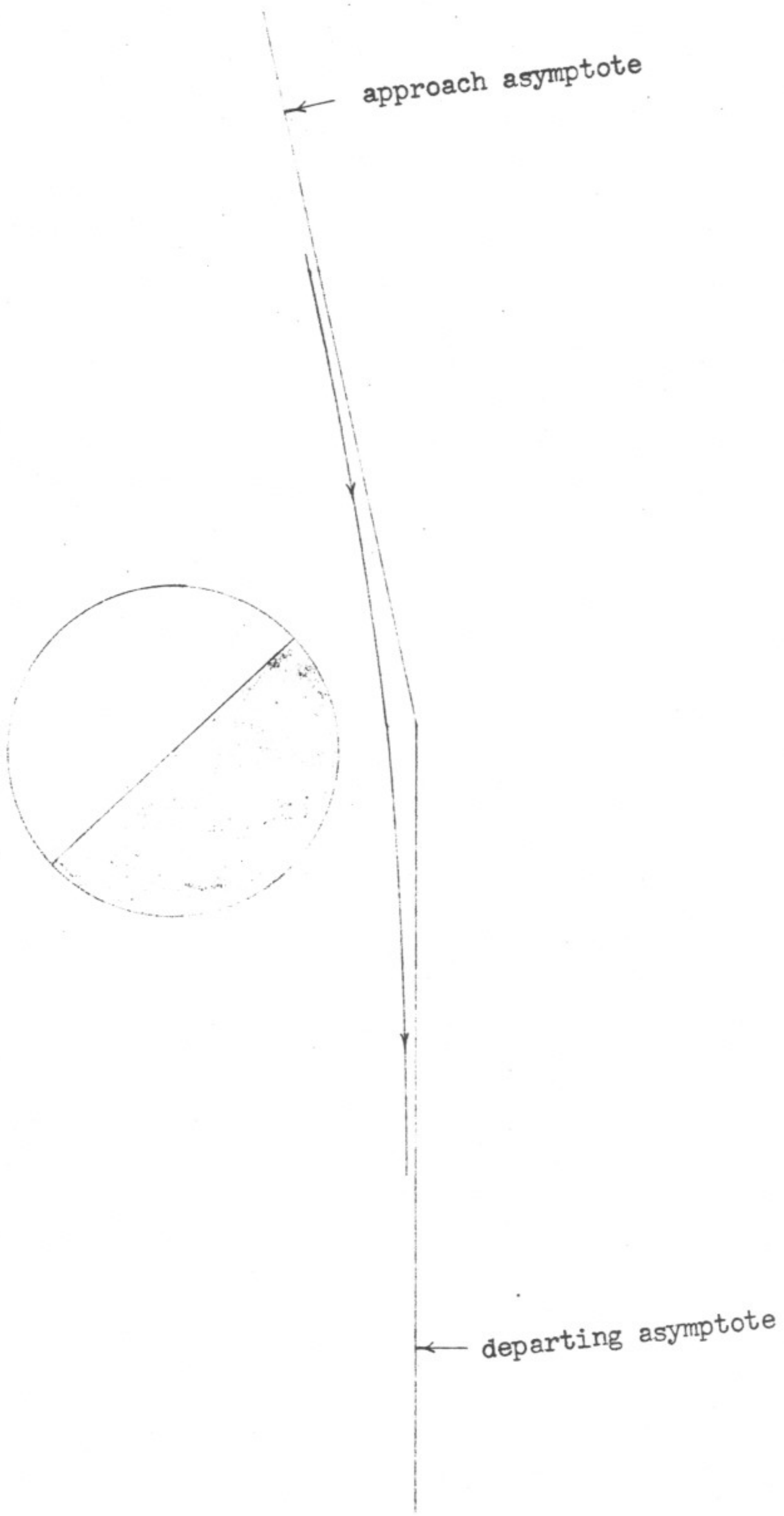


Figure 36

When more attention is directed towards manned interplanetary flights of the near future the above reconnaissance trajectories of the 1970 and 1972 launch periods should warrant serious consideration.

d. Manned Landings on Venus and Mars Utilizing Advanced Trajectories

We shall now give another example of how advanced trajectories can be applied to early manned interplanetary missions. In particular let us consider the trajectories for manned landing on venus and mars. Tables 17 and 18 describes the optimum venus-earth and mars-earth transfer trajectories respectively. If we compare these tables with tables 1 and 3 we find that the optimum venus-earth return trajectories occure 20 to 40 days after the optimum earth-venus departing trajectories. The optimum mars-earth return trajectories occure 40 to 60 *before* the optimum earth-mars departing trajectories!

A brief study of manned missions to venus and mars during 1970-1975 was carried out to obtain some characteristic properties of the most favorable trajectory profiles. The results appear in Tables 19 and 20. The symbol ΔT represents the number of days spent on venus or mars. One immediately notices the unusually high hyperbolic excess velocities associated with the return mars-earth trajectories. This is a characteristic property of all missions to mars of this type where only one vehicle is involved. These high velocities are due to the fact that the mars launch date will always occure much later than the optimum launch dates for the mars-earth transfers. Thus in general only a relatively small amount of time can be devoted to the exploration of mars. These properties are also generally true for the mission to venus. It is important to note however that it is not always advantageous from an energy point of view to start the return trip as soon as possible. For example the venus launch energies for Type II venus-earth trajectories actually decrease from Dec. 22, 1970 to Jan. 15, 1971 although the optimum launch date is Oct. 21, 1970. This situation does not show up for the mars missions. Type I venus-earth and mars-earth transfers require much higher launch energies than the

TABLE 17

Some Important Properties of Optimum Venus-Earth Transfer Trajectories

TYPE I

Launch Period	Min. HEV_1	Max. HEV_1	Min. T_{12}	Max. T_{12}	Min. HEV_2	Max. HEV_2
8/6/70 - 10/17/70	3.21	6.15	94	122	3.65	5.87
3/24/72 - 5/21/72	4.04	6.32	92	128	3.27	7.91
11/6/73 - 1/15/74	4.12	6.38	96	162	2.71	7.60

TYPE II

Launch Period	Min. HEV_1	Max. HEV_1	Min. T_{12}	Max. T_{12}	Min. HEV_2	Max. HEV_2
7/15/70 - 3/22/71	3.47	5.93	152	348	2.85	10.48
3/8/72 - 10/20/72	3.13	5.78	174	348	3.04	9.77
10/17/73 - 5/9/74	2.96	5/58	166	330	3.59	10.04

TABLE 18

Some Important Properties of Optimum Mars-Earth Transfer Trajectories

TYPE I

Launch Period	Min. HEV_1	Max. HEV_1	Min. T_{12}	Max. T_{12}	Min. HEV_2	Max. HEV_2
7/7/70 - 7/8/71	2.55	5.57	172	360	3.05	9.02
12/21/72 - 8/14/73	3.53	5.55	168	296	2.93	4.93

TYPE II

Launch Period	Min. HEV_1	Max. HEV_1	Min. T_{12}	Max. T_{12}	Min. HEV_1	Max. HEV_2
3/3/70 - 9/22/71	2.75	5.48	242	508	3.45	9.00
3/22/72 - 11/20/73	3.71	5.61	260	516	3.00	11.56

TABLE 19

Near Optimum Trajectory Profiles for Manned Exploration of Venus

Launch Date	HEV_1	T_{12}	HEV_2	ΔT	Venus Launch Date	HEV_2	T_{23}	HEV_3	Arrival Date	Total Time
Aug. 18 1970	2.91	116	5.43	32	Jan. 13 1971	4.81	302	8.65	Nov. 11 1971	450
Mar. 26 1972	3.50	112	6.16	20	Aug. 5 1972	4.59	284	8.51	May 16 1973	416
Nov. 10 1973	3.66	106	4.89	14	Mar. 10 1974	4.49	284	8.34	Dec. 19 1974	404

TABLE 20

Near Optimum Trajectory Profiles for Manned Exploration of Mars

Launch Date	HEV_1	T_{12}	HEV_2	ΔT	Mars Launch Date	HEV_2	T_{23}	HEV_3	Arrival Date	Total Time
May 19 1971	3.53	135	5.52	9	Oct. 10 1971	5.78	264	9.86	June 30 1972	408
July 27 1973	3.90	175	3.62	9	Jan. 27 1974	6.81	258	15.77	Oct. 12 1974	442

TABLE 21

Alternative Trajectory Profiles for Manned Exploration of Mars

Launch Date	HEV_1	T_{12}	HEV_2	ΔT	Mars Launch Date	HEV_2	T_{23}	HEV_3	Arrival Date	Total Time
Aug. 12 1970	3.26	309	6.75	19	July 6 1971	4.19	246	6.44	Mar. 8 1972	574
June 4 1972	4.32	335	6.19	61	June 19 1973	3.54	190	3.42	Dec. 26 1973	586

Type II return trajectories and hence are not employed.

Now it may appear that the manned exploration of mars will require more powerful launch vehicles than those required for the venus missions but since the escape velocities of venus and mars are 10.40 and 5.03 km/sec respectively, the missions to venus will require more powerful rockets. In view of this low mars escape velocity it appears that if we can find low earth launch energy trajectories which will take a vehicle to mars such that the mars arrival date is sufficiently close to the favorable mars-earth launch dates it may be possible to conduct manned missions to mars before nuclear rockets become available. Such trajectories do indeed exist. As a matter of fact we have already calculated some. These are the earth-venus-mars trajectories. By utilizing these trajectories as our earth to mars transfer trajectories it may be possible to save a great deal of energy. Table 21 contains trajectory profiles of the 1971 and 1973 manned missions to mars using earth-venus-mars trajectories as the earth to mars transfers. These trajectories clearly require far less weight lifting capabilities of the primary earth launch vehicle. Unfortunately the earth-venus-mars trajectories of 1973, and 1975 require much longer flight times and hence are impractical earth to mars transfer trajectories.

Now there is still another way which could bring about early manned voyages to mars. This method will require even less energy than the previous methods. In order that this method be clearly understood we shall consider an actual example. Let A represent an earth-venus-mars-earth manned reconnaissance vehicle of the type described above. We shall launch this vehicle on the July 19, 1970 trajectory of Table 15. This vehicle will be manned by a skeleton crew of only one or two persons but shall carry a standard earth landing module which accommodates a regular sized crew of 5 or 6 astronauts. Let B denote a second vehicle similar to A but carrying in place of the earth landing module a mars landing module which is very similar to the lunar excursion module of the Apollo mission. This vehicle is launched on the August 22, earth-venus-mars trajectory of Table 12. It shall be manned by a crew of 3 or 4. This flight will take 305 days and terminates in the vicinity of mars where the crew abandons the vehicle and decends to the surface of mars in the small mars excursion module. Thus the landing occures on June 23, 1971 which is 16 days before vehicle A makes its mars closest approach. Consequently after 16 days of exploration on mars the crew of the second vehicle launches its mars excursion module on July 9 and rendezvous with A which will be making its closest approach. The excursion module is then **abandoned** and the remaining voyage proceeds just as in the case of earth-venus-mars-earth reconnaissance missions.

Let us now perform a rough energy analysis of this particular mission profile and compare it to the direct ascent moon mission to be carried out with the Nova. Now it is clear that the vehicles A and B will have to carry much more life support equipment than the moon vehicle. The first vehicle will have to carry enough provisions to last 1 or 2 crew members 649 days **plus** an extra supply to last the crew of the second vehicle 294 days. The second vehicle however need carry only enough provisions to last its crew 321 days. It is important to notice that unlike the moon vehicle, A does not have to carry any other landing module other than its earth return module which in fact may be very similar to the landing module of the moon vehicle. The second vehicle need not carry any earth return module.

We recall that the characteristic velocity of any mission profile is the sum total of all the velocities that a rocket has to develop or dissipate by fuel consumption during the entire course of the mission. Consequently since the earth's escape velocity is 11.19 km/sec we find from Table 15 that the characteristic velocity V_{CA} of A's mission is

$$V_{CA} = \sqrt{11.19^2 + 3.63^2} + \sqrt{8.41^2 + 11.19^2} = 25.76 \text{ km/sec}$$

From tables 12 and 15 it follows that the characteristic velocity V_{CB} of B's mission is

$$V_{CB} = \sqrt{11.19^2 + 3.47^2} + \sqrt{6.46^2 + 5.03^2} + \sqrt{5.03^2 + 5.61^2} = 27.44 \text{ km/se}$$

The minimum velocity required to reach the neutral point between the earth and moon is 11.1 km/sec. Thus since the moon's escape velocity is 2.34 km/sec, the characteristic velocity V_{CM} of the moon mission is

$$V_{CM} = 11.1 + 2.34 + 2.34 + 11.1 = 26.88 \text{ km/sec}$$

Consequently our calculations show that the characteristic velocity required for A's mission is actually less than that required for the moon mission while the characteristic velocity of B's mission is only .56 km/sec greater.

Let us assume for simplicity that these three rockets have only one stage. Consider the well known formula

$$\frac{\text{initial mass of rocket}}{\text{final mass of rocket after burning fuel}} = e^{\frac{V}{c}}$$

where V is the characteristic velocity for a mission using a single stage rocket with exhaust velocity equal to c. Let M_{AP}, M_{BP} and M_{MP} denote the payloads of A, B and the moon rocket respectively where we take payload to be the total mass of the rocket minus its fuel load. Consequently if all three of the rockets have identical initial masses we obtain

$$M_{AP} = 1.37 \, M_{MP}$$

$$M_{BP} = .852 \, M_{MP}$$

Before commenting on the implications of these simple calculations let us consider a corresponding trajectory profile for the manned mission to mars for the 1972 earth-venus-mars-earth launch period. In the case of the 1970 mission it was possible to choose a near optimum trajectory profile (here an optimum trajectory is one which yields minimum characteristic velocity) from those trajectories given in tables 12 and 15. This situation is quite uncommon since minimum launch energies do not necessarily imply minimum approach hyperbolic excess velocities. Consequently the following profile which we shall choose for the 1972 mission was obtained from the fine net calculations of the 1972 earth-venus-mars-earth and earth-venus-mars trajectories which do not appear in tables 13 and 16.

As in the case of the 1970 mission we let A denote the vehicle traveling on the free fall earth-venus-mars-earth trajectory which will rendezvous with the mars excursion module of the second vehicle denoted by B. Unlike the example given for the 1970 mission we shall launch B before A. In particular we shall launch it on May 31, 1972 on an earth-venus-mars trajectory which will take it past venus on Nov. 17, 1972 with a distance of closest approach of 9223. km. It will reach the vicinity of mars on May 12, 1973 after a total flight time of 346 days. The vehicle's departing hyperbolic excess velocity will be 4.27 km/sec and will approach mars with a hyperbolic excess velocity of 6.03 km/sec. Four days after launching B, A is launched. Its earth-venus-mars-earth trajectory will take it past venus on Nov. 19, 1972 with a distance of closest approach of 9164 km. This vehicle will pass mars on May 23, 1973 whereupon it will rendezvous with the excursion module of B and take its crew back to earth on Oct. 17, 1973. This vehicle will leave earth with a hyperbolic excess velocity of 4.33 km/sec and after a total flight time of 499 days it will return with a hyperbolic excess velocity of 9.51 km/sec. This mission will allow the crew of B to spend 11 days of exploration on mars. This mission profile is 146 days shorter than the manned

1970 mars mission. Thus A need carry only enough provisions and life support equipment to last 1 or 2 men 499 days plus an additional supply to last the crew members of B only 147 days although B must have an initial supply to last its crew 346 + 11 or 357 days.

The characteristic velocities associated with this mission profile are found to be

$$V_{CA} = \sqrt{11.19^2 + 4.33^2} + \sqrt{9.51^2 + 11.19^2} = 26.67 \text{ km/sec}$$

$$V_{CB} = \sqrt{11.19^2 + 4.27^2} + \sqrt{6.03^2 + 5.03^2} + \sqrt{5.97^2 + 5.03^2} = 27.62 \text{ km/se}$$

Thus we again find that the characteristic velocities required for the missions of A and B are almost equal to that of the manned moon mission. If we again assume our rockets to be single stage we find

$$M_{AP} = 1.06\ M_{MP}$$

$$M_{BP} = .815\ M_{MP}$$

Let us now carry these calculations one step further and see if the missions could be actually achieved by a Nova type vehicle. Table 22 contains a list of these launch vehicles along with their expected design capabilities.

TABLE 22

Escape Payloads of Nova Launch Vehicles

Launch Vehicle	Escape Payloads	Possible First Use
Small Nova	150 k	1968-9
Nova	270 k	1970
Advanced Nova	450 k	1970-2
Post Nova	1,060 k	?

Suppose the re-entry weight of the earth return modules is 25,000 lbs and that $\frac{2}{3}$ of its remaining energy can be dissipated in the earth's atmosphere. Now for the 1970 mission profile this means that

$$\sqrt{\frac{1}{3}\left(8.41^2 + 11.19^2\right)} \qquad \text{km/sec}$$

must be eliminated by a retro rocket. Denoting this velocity by ΔV_{oo} and assuming that the rocket's exhaust velocity is 3.6 km/sec we find that the vehicles mass M_{oo} before retro must be approximately

$$M_{oo} = 25e^{\frac{\Delta V_{oo}}{3.6}} \text{ k} \quad = 235 \text{ k}$$

Since the Nova launch vehicle described in Table 22 will have the capability of sending a payload weight of approximately

$$270\, e^{\frac{1}{3.6}\left(11.19 - \sqrt{11.19^2 + 3.63^2}\right)} \text{ k} = 230 \text{ k}$$

on the escape trajectory required for A's mission this launch vehicle could not be used. On the other hand the advanced Nova will have the capability of sending approximately 383 k on the required escape trajectory. This means that the primary crew compartment containing the provisions and life support equipment could weigh as much as 148 k. This figure is quite realistic hence we conclude that if the return module could shed $\frac{2}{3}$ of its approach energy in the earth's atmosphere the advanced Nova could be employed as A's launch vehicle.

Let us now see if this launch vehicle could also be used for B's mission. Suppose that the mars excursion module at the moment it rendezvouses with A to begin the voyage back to earth weighs 15 k. It's launch weight would then be approximately

$$15e^{\frac{1}{3.6}\sqrt{5.03^2 + 5.61^2}} = 120 \text{ k}$$

Suppose the provisions and life support equipment necessary to spend 16 days on mars weight $2\frac{1}{2}$ tons. Then at the moment the excursion vehicle arrives on mars it will weigh 125 k. Thus before retro it will weigh about

$$125\, e^{\frac{1}{3.6}\sqrt{6.46^2 + 5.03^2}} = 1{,}210 \text{ k}$$

If we assume that the primary crew compartment along with the life support equipment weighs 150 k the main vehicle leaving earth must weigh 1360 k of which 1190 k would be fuel for the mars excursion module. Now the advanced Nova will be capable of sending approximately 390 k on B's escape trajectory. Hence this vehicle plus 220 k of fuel for the mars excursion module could be launched by the advanced Nova. The remaining 970 k of fuel could then be supplied by injecting two orbiting tankers into the required trajectories which would enable them to rendezvous with B and transfer their fuel loads. Thus the entire mission could be carried out by employing two advanced Novas along with two orbiting tanker vehicles. The same launch vehicles could also be used for the 1972 mission.

Since the nuclear rocket engines required for the conventional earth-mars, mars-earth manned mission profiles will probably not become operational before 1980 the above calculations show that it is not necessary to await the development of these advanced propulsion systems before carrying out manned missions to mars. Consequently the development of this rocket should proceed on a schedule which would enable it to be used for the 1972 earth-venus-mars-earth manned reconnaissance mission or better yet for the manned mission to mars. If manned interplanetary missions are not carried out before 1977 they may have to be postponed until after 1981 because the sun will be very active during this time interval.

The unusual mission profiles described above for the earth manned missions to mars leads us to the last major topic of this paper.

C. Interplanetary Transportation Networks to Support Manned Bases On Venus and Mars

When man first ventures out into interplanetary space his voyages will in essence be very similar to the great sea voyages of the 15'th and early 16'th centuries. Thus in a sense it will be almost like history repeating itself although in this case the principal competing countries will be the United States and Russia instead of England and Spain.

After the first flights to mars and venus, man will naturally construct bases on these planets (by utilizing 8 or 10 Novas it may even be possible to construct a temporary base on mars as early as 1971 or 1973). These bases no matter when they are constructed will naturally require a means by which men and equipment can be taken to and from these bases. Now in the distant future when propulsion systems far more advanced than those currently being studied are developed it will probably be possible to make interplanetary voyages such as earth-mars transfers with flight times as short as one or two weeks. But for the near future all interplanetary transfers will have to be made on near optimum transfer trajectories with low departure and arrival hyperbolic excess velocities. Consequently a great deal of life support equipment will be necessary to transport a few persons from one planet to another. In short, cargo vehicles shall probably be robot type vehicles carrying no equipment necessary for manned flight while the manned vehicles will probably carry very limited amounts of cargo. In addition to carrying all the extra equipment for manned flight these vehicles will also probably be required to be able to induce some artificial gravity. Hence the transportation of just 10 men for example from mars to earth should ordinarily require a fairly large and very expensive rocket. Methods of recovery will become a necessity. This problem of economics can be conveniently solved by constructing a long lasting interplanetary transportation network designed for the sole purpose of transporting personnel from one planet to another. Since the traffic involving venus, earth and mars will undoubtedly be the heaviest this network can be specifically designed to provide transportation for only these three planets.

The basic underlying idea behind the system which we shall now consider rests on the potentialities of extremely accurate planetary approach guidance equipment. If a vehicle, equipped with such a system can be made to have an indefinite operating life time, it can be launched on a free fall trajectory of the form

$$P_1 - P_2 - P_3 \dots$$

which will permit the vehicle to rendezvous with any planet of the solar system an indefinite number of times with the expenditure of almost zero energy. Preliminary calculations show that if the planets P_i are restricted to venus, earth and mars where P_1 = earth and $P_i \neq P_{i+1}$ for all i, it is possible to find sequences $\left\{P_1, P_2, P_3, \dots, P_n\right\}$ such that the flight times $T_{i+1} - T_i$ $(i = 1, \dots n-1)$ from P_i to P_{i+1} are compareable to those required for optimum $P_i - P_{i+1}$ transfers leaving P_i at T_i. Indeed in many cases these flight times are often shorter than those of the optimum transfers. Moreover some of these trajectories were found to have very low launch energies.

The network can be established by first constructing many large space vehicles which are to be used in the network. This can be done by methods of prefabrication and orbital assembly. These vehicles can be designed to accommodate 20 to 60 persons and since artificial gravity will be highly desirable the geometry of the vehicles could be a torus with an outside diameter of perhaps 200 to 300 feet. When each individual vehicle is completed one simply awaits its launch date T_1 when the vehicle (i.e., space bus) is injected into its interplanetary trajectory. This could be accomplished by convient strap-on solid propellant rocket engines. The vehicle will carry extra provisions and life support equipment to last until it makes its first earth rendezvous whereupon its supply of provisions and life support equipment can be replenished to last until it makes its second earth rendezvous, etc. This transfer vehicle shall also be provided with standard planetary excursion modules which can be left behind on a planet with its occupants to be used by other astronoughts to rendezvous with another space bus of the network which will take them back to earth or to another planet. These small excursion modules need carry very little life support equipment and hence can have relatively high mass ratios.

Our network thus consists of perhaps 20 or 30 large transfer vehicles moving along different free fall trajectories which continually rendezvouses with venus, earth and mars. The vehicles carry standard planetary excursion modules which will permit some of their occupants, to leave a vehicle and descend to the surface of a passing planet without ever having to alter the course of the transfer vehicle. They are also equipped with docking facilities for excursion modules launched from a passing planet by occupants wishing to go to the next planet which the transfer vehicle rendezvouses with.

When each transfer vehicle is injected into its particular interplanetary trajectory a large unmanned robat vehicle carrying a great amount of fuel for the excursion modules can also be injected a few seconds later onto an almost identical trajectory. This second vehicle could then always follow the transfer vehicle and can be used to refuel the excursion modules. This second tanker vehicle could itself be refueled during the earth approaches.

The transportation network outlined above seems attractive for several reasons. One notices that all the vehicles involved in the network can be used as often as desired. The transfer vehicles could be very large and hence could be designed so that these necessarily long planetary transfers can become quite acceptable. The relatively large number of transfer vehicles in the network would greatly preclude the possibility of a crew running out of supplies and becomming isolated on venus or mars. Finally the economics and up keep of the network seems radily attainable.

Since these advanced free fall trajectories were not studied deeply enough to warrant a detailed report, we shall not give any numerical results at this time.

V. CONCLUDING REMARKS

The numerical results described in Section IV show that advanced free fall interplanetary trajectories involving more than two planets of the form

$$P_1 - P_2 - P_3 - \cdot \cdot \cdot \cdot - P_n$$

can be of great value in both the unmanned and manned exploration of the inner planets. Our results definitely indicate that these trajectories should no longer be viewed as an academic curiosity. Let us take another good look at the difficulties which we are forced to contend with if we insist on using the conventional direct flight transfer trajectories. We have found that the four principal time - energy barriers were:

1. The lowest possible launch energies for earth - Mercury missions are exceptionally high.
2. Direct flight earth - Mars missions have their favorable launch periods separated by long intervals of time lasting approximately 25.6 months.
3. Minimum energy manned Mars reconnaissance trajectories of the form earth-Mars-earth require flight times in excess of 1000 days.
4. The favorable Mars - earth launch periods occurs 40 to 60 days _after_ the favorable earth-Mars launch periods resulting in very high hyperbolic excess velocities for all trajectories of manned Mars missions.

It is because of these irrevocable hard facts that the National Aeronautics and Space Administration's timetable for many interplanetary missions is so modest. This is especially true for the manned reconnaissance and landing missions to Mars.

With the aid of advanced trajectories a great part of NASA's modest timetable (which was oriented to fit the uncertain timetable for nuclear propulsion) can be completely (and perhaps even more realistically) revised. The development of accurate planetary approach guidance does not require any new scientific breakthroughs.

The most important interplanetary mission effected by the application of advanced trajectories is, of course, the manned mission to Mars. We have seen that if the advanced Nova launch vehicle can be made man rated by 1970, it is entirely possible to begin a manned mission to Mars the same year. Since this launch vehicle will most probably be man rated by 1972, the manned mission to Mars beginning in that year could be undertaken. These missions would, of course, render the manned Mars reconnaissance mission unnecessary. If these manned missions to Mars are not attempted, then the Nova launch vehicle will not be capable of making these missions possible until 1977. But since manned interplanetary flights during 1977 to 1981 may be very hazardous because of high solar activity it may not be until after 1981 that these missions could be undertaken.

In concluding this paper the author is convinced that the numerical results of Section IV definitely warrants the rapid development of

1. Highly accurate and reliable planetary approach guidance systems.

The real possibility of using the advanced Nova as the primary launch vehicle for the manned Earth-Venus-Mars-Earth reconnaissance missions on the first manned landings on Mars for the 1970 and 1972 launch periods indicates that

2. The development of the Advanced Nova should proceed with all possible urgency

along with

3. The development of a Mars excursion module to be used with the Advanced Nova launch vehicle.

These proposals of course presupposes that a much more detailed study will back up the rather brief analysis given in this paper.

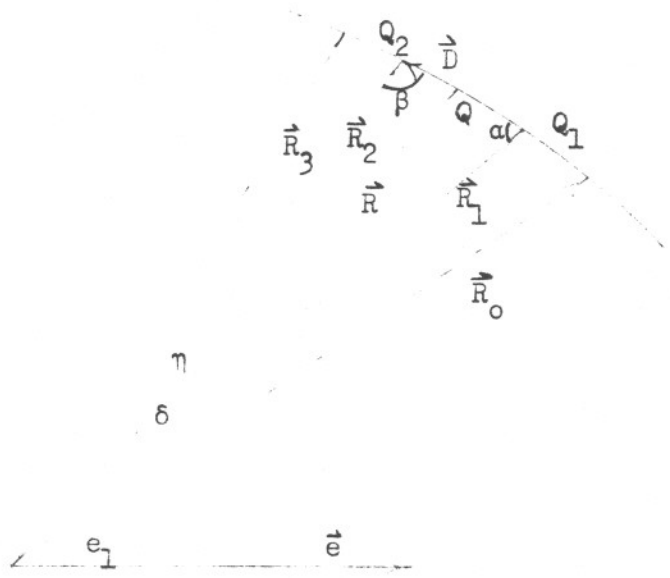
If the above three programs are begun or stepped up during this relatively early period, the United States could truly set a pace in the exploration of interplanetary space that will be very difficult if not impossible to follow.

Appendix 1

The Calculation of Planetary Position and Velocity Vectors

In section 2 we have developed a very convenient calculus for dealing with conic trajectories in three dimensions. We have seen that it is ideally suited for problems in astronautics. Thus as one might suspect it can be easily employed to calculate planetary position and velocity vectors for any time t. We shall now give the method which was actually used for the numerical computation of the above trajectories. It has been observed that the accuracy obtained by this method approaches that obtained by the most involved and complicated techniques of classical celestial mechanics.

Let $\vec{R}$ be the position vector of a particular planet at the time t which is to be calculated. Suppose $\vec{R}_1$ and $\vec{R}_2$ are two position vectors of the planet given in a planetary ephemeris (see 4) corresponding to t_1 and t_2 respectively where $t_1 \le t \le t_2$. We shall take $t_2 - t_1 = 10$ days for the first six planets. Missions to Uranus, Neptune and Pluto take too long for elliptic transfer trajectories.



In the above figure the vectors $\vec{R}_o$ and $\vec{R}_3$ correspond to the planets position vectors at t_o and t_3 respectively which are given as near-by entries in the planetary ephemeris where $t_o < t_1 \leq t \leq t_2 < t_3$. These vectors will be used to calculate the planets osculating elliptical orbit at the time t. This calculated orbit will not be the true osculating orbit at time t because the planets actual path about the sun is not elliptical. However if the arc subtended by $\vec{R}_o$ and $\vec{R}_3$ is not too large it may be assumed to be elliptical. This assumption applies even better to the arc subtended by $\vec{R}_1$ and $\vec{R}_2$ but $\sphericalangle$ $\vec{R}_1\vec{R}_2$, being small for most planets, will magnify errors in $\vec{R}_1$, $\vec{R}_2$ appearing in the ephemeris. Thus if we choose $t_1 - t_o = t_3 - t_2 = 10$ k days where k is an integer equal to the k'th planet of the solar system which is being considered, both conditions can be partially satisfied. Hence one easily finds

Planet	k	$t_3 - t_o$
Mercury	1	30 days
Venus	2	50 "
Earth	3	70 "
Mars	4	90 "

The osculating orbit is then calculated by first finding the semi-major axis by means of Lambert's Theorem in the form of (24) where $t = t_3 - t_o$. The eccentricity is then calculated with the aid of (30). The E and H vectors may now be calculated by means of (8) - (12).

In the figure $\vec{D} = \vec{R}_2 - \vec{R}_1$. Hence $\vec{D}$ lies in planets plane of motion and intersects the vector $\vec{R}$ at some point Q. The points Q_1 and Q_2 are the positions of the planet at t_1 and t_2 respectively. If $d = \overline{Q_1Q}$ it is easy to see that

$$\hat{R} = \frac{\vec{R}_1 + f\vec{D}}{|\vec{R}_1 + f\vec{D}|} \qquad \text{(i)}$$

where $f = \frac{d}{D}$. It is also evident from the figure that the following trigonometric relations are true:

$$\frac{d}{\sin\delta} = \frac{R_1}{\sin(\beta+\eta)}$$

$$\frac{D-d}{\sin\eta} = \frac{R_2}{\sin(\beta+\eta)}$$

Consequently it follows that

$$f = \frac{1}{1 + \frac{R_2 \sin(\sigma-\delta)}{R_1 \sin\delta}} \qquad \text{(ii)}$$

where $\sigma = \delta + \eta$. This angle σ may easily be calculated by

$$\sigma = \cos^{-1}\left(\frac{\vec{R}_1 \cdot \vec{R}_2}{R_1 R_2}\right)$$

Now the angle δ is dependent on some scalar function $F(t)$. Since $t_2 - t_1 = 10$ days is small compared to any of the planets periods about the sun, $F(t)$ may be expressed as a rapidly converging Taylor series about t_1.

$$\delta = F(t) - F(t_1) + \frac{d\,F(t_1)}{dt}(t-t_1) + \frac{1}{2!}\frac{d^2F(t_1)}{dt^2}(t-t_1)^2 + \ldots \qquad \text{(iii)}$$

Since $\delta = 0$ at $t = t_1$, $F(t_1) = 0$. Let $\theta = \sphericalangle \vec{e}\,\vec{R}$ and suppose that we refer time to the last perihelion passage so that $\theta = 0$ at $t = 0$. Then

$$\delta = F(t) = \theta - \theta_1$$

and it follows in view of (34) that

$$\frac{dF(t_1)}{dt} = \frac{h}{\ell^2}\,(1 + e \cos \theta_1)^2$$

$$\frac{d^2F(t_1)}{dt^2} = -2\left(\frac{h}{\ell^2}\right)^2 e(1 + e \cos \theta_1)^3 \sin \theta_1$$

$$\frac{d^3F(t_1)}{dt^3} = 2\left(\frac{h}{\ell^2}\right)^3 e(1 + e \cos \theta_1)^4 \left[3e \sin^2 \theta_1 - (1 + e \cos \theta_1) \cos \theta_1\right]$$

For all planets except Mercury the Taylor series converges very rapidly so that all terms except the first three can be neglected. In the case of Mercury the fourth term does contribute a small but sometimes a non-negligible amount to the series. The maximum contribution of this term has been found to be about .1 degrees. Thus for the fourth term one finds

$$\frac{d^4F(t_1)}{dt^4} = 2e\left(\frac{h}{\ell^2}\right)^4 \sin \theta_1 (1 + e \cos \theta_1)^5 (24\, e^2 \cos^2 \theta_1 + 13\, e \cos \theta_1 - 12\, e^2 + 1)$$

The values for cos θ_1 and sin θ_1 may be computed by

$$\cos \theta_1 = \frac{\vec{R}_1 \cdot \vec{e}}{R_1 e}$$

$$\sin \theta_1 = S \sqrt{1 - \cos^2 \theta_1}$$

where S is equal to the sign of sin θ_1 and is easily seen to be

$$S = \frac{\vec{e} \times \vec{R}_1 \cdot \vec{h}}{|\vec{e} \times \vec{R}_1 \cdot \vec{h}|}$$

where $\vec{h}$ is the osculating H - vector. Thus after calculating each term δ may be obtained by (iii). Upon substituting this value into (ii) and the resulting value of f into (i), R can be calculated. The desired position vector of the planet at the time t is obtained by

$$\vec{R} = \frac{\ell}{1 + \vec{e} \cdot \hat{R}} \hat{R}$$

which follows from (7). The planets velocity vector $\vec{V}$ is immediately calculated by

$$\vec{V} = \frac{1}{\ell} \vec{h} \times (\hat{R} + \vec{e})$$

APPENDIX 2

Constants of the Solar System Used in the Calculations

The following constants have been employed throughout all the numerical computation:

$$\text{astronomical unit} = 1.495990 \times 10^{8} \text{ km}$$

$$GM_{\text{sun}} = \mu_{\text{sun}} = 2.9591221 \times 10^{-4} \ \frac{\text{a. }\mu.^{3}}{\text{day}^{2}}$$

$$\text{Mean Obliquity (for 1950)} = 23^{\circ}\ 26'\ 44.84''$$

Constants used concerning the planets are given in the following table where R = radius (in km), m = planets mass, M = sun's mass and $\mu = Gm$ (in $a\,\mu^{3}/\text{day}^{2}$). The radius given for Venus includes the visible atmosphere. Thus the distance of closest approach to Venus given in the numerical tables refers to the distance of closest approach to the planet's cloud layer and not its actual surface.

PLANET	R	$\left(\frac{m}{M}\right)^{\frac{2}{5}}$	μ
Mercury	2330.0	.00193138	4.835167×10^{-11}
Venus	6100.0	.00570377	7.241303×10^{-10}
Earth	6378.2	.00617728	8.887552×10^{-10}
Mars	3415.0	.00253523	9.582649×10^{-11}

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